Timeless system, superluminal phenomena, dark matter and Big Bang[1]

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Abstract. — With the notion of flavor-oscillation clock of Ahluwalia et. al. or the notion of local time introduced by the author, we will consider a quantum mechanical local system that forms a bound state with respect to the local Hamiltonian of the system. We will see that such a local system is considered timeless and hence the principles of relativity do not apply to those timeless systems. As application, we will suggest a possibility of the faster-than-light phenomena and a possible explanation of dark matter and Big Bang phenomenon consistent with the theory of relativity and the conservation law of matter.

1 Definition of local time

Ahluwalia et. al. in [1], [2], [3] introduced the concept of flavor-oscillation clock of a quantum mechanical system, which was later noticed in [4] to be the same as the notion of local clock introduced earlier by the author in [5] and developed further in [6], [7], [8], [9] and other papers in the references. We here review some basic ideas considered there.

A quantum mechanical many body system with Hamiltonian $H_{n\ell}$ on a Hilbert space $\mathcal{H}_{n\ell} = L^2(\mathbb{R}^{3n})$ is called a local system and is denoted $(H_{n\ell}, \mathcal{H}_{n\ell})$, where $N = n + 1$ is the number of particles in the system with $n \geq 1$, and the label $\ell = 1, 2, \ldots$ distinguishes different systems with the same number of particles.

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Concretely stating, $H_{n\ell}$ is defined from the Hamiltonian $\tilde{H}_{n\ell}$ of N-particles located at $X_1, \ldots, X_N$ ($X_j = (X_{j1}, X_{j2}, X_{j3}) \in \mathbb{R}^3$) in a given Euclidean space $\mathbb{R}^3$. The Hamiltonian $\tilde{H}_{n\ell}$ is defined in $L^2(\mathbb{R}^{3N})$ and given by

$$\tilde{H}_{n\ell} = -\hbar^2 \sum_{j=1}^{N} \frac{1}{2m_j} \frac{\partial^2}{\partial X_j^2} + \sum_{1 \le i < j \le N} V_{ij}(X_j - X_i), \tag{1}$$

where $\hbar = h/(2\pi)$ with h being Planck constant, $m_j > 0$ is the mass of the j-th particle,

$$\frac{\partial^2}{\partial X_j^2} = \sum_{k=1}^{3} \frac{\partial^2}{\partial X_{jk}^2}$$

is Laplacian in $\mathbb{R}^3$, and $V_{ij}(x)$ ($x \in \mathbb{R}^3$) is a real-valued pair potential which describes the interaction between the particles i and j. Henceforth we assume that we take a unit system in which $\hbar = 1$ so that we will consider

$$\tilde{H}_{n\ell} = -\sum_{j=1}^{N} \frac{1}{2m_j} \frac{\partial^2}{\partial X_j^2} + \sum_{1 \le i < j \le N} V_{ij}(X_j - X_i) \tag{2}$$

instead of (1).

As the interaction depends just on relative coordinate $X_j - X_i \in \mathbb{R}^3$ of the particles, we can remove the center of mass

$$X_C = \frac{m_1 X_1 + \cdots + m_N X_N}{m_1 + \cdots + m_N}$$

from the Hamiltonian $\tilde{H}_{n\ell}$ by introducing a relative coordinate system

$$x_i = X_{i+1} - \frac{m_1 X_1 + \cdots + m_i X_i}{m_1 + \cdots + m_i} \qquad (i = 1, 2, \ldots, n, n = N - 1),$$

and obtain the following Hamiltonian in $\mathcal{H}_{n\ell} = L^2(\mathbb{R}^{3n})$ which describes the internal motion of the local system $(H_{n\ell}, \mathcal{H}_{n\ell})$.

$$H_{n\ell} = -\sum_{k=1}^{n} \frac{1}{2\mu_k} \frac{\partial^2}{\partial x_k^2} + \sum_{1 \le i < j \le N} V_{ij}(x_{ij}). \tag{3}$$

Here the relative coordinate $x_{ij} = X_j - X_i$ is the one expressed by the new coordinate x_i ($i = 1, 2, \ldots, n$) introduced above and μ_k is the reduced mass defined by

$$\frac{1}{\mu_k} = \frac{1}{m_{k+1}} + \frac{1}{m_1 + \cdots + m_k} \qquad (k = 1, 2, \ldots, n).$$

The remaining Hamiltonian of the center of mass

$$H_C = -\frac{1}{2\sum_{j=1}^{N} m_j} \frac{\partial^2}{\partial X_C^2}$$

is a constant multiple of the negative Laplacian $-\Delta$ in $\mathbb{R}^3$ and could be omitted when considering the relative motion inside the system. We will actually do this later when we make the postulates of relativity on the motion of centers of mass of various local systems. By this change of coordinate the original Hamiltonian $\tilde{H}_{n\ell}$ is equal to the Hamiltonian

$$\tilde{H}_{n\ell} = H_C \otimes I + I \otimes H_{n\ell}$$

defined in the tensor product $L^2(\mathbb{R}^{3N}) = L^2(\mathbb{R}^3) \otimes L^2(\mathbb{R}^{3n}) = L^2(\mathbb{R}^3) \otimes \mathcal{H}_{n\ell}$.

Thus far we have used only the position operator $x_j = (x_{j1}, x_{j2}, x_{j3})$ and the conjugate momentum operator $p_j = (p_{j1}, p_{j2}, p_{j3}) = -i\partial/\partial x_j = -i\partial_{x_j} = -i(\partial/\partial x_{j1}, \partial/\partial x_{j2}, \partial/\partial x_{j3})$ to describe the local system $(H_{n\ell}, \mathcal{H}_{n\ell})$. We now introduce the concept of time as follows in this system. Namely the unitary group $\exp(-itH_{n\ell})$ generated by the local Hamiltonian $H_{n\ell}$ is called the flavor-oscillation clock, or local clock of the local system $(H_{n\ell}, \mathcal{H}_{n\ell})$, and the parameter t in the exponent of the local clock $\exp(-itH_{n\ell})$ is called the local time of the system.

We here recall that the closed space spanned by bound states of $H_{n\ell}$ is called a pure point spectral subspace of $H_{n\ell}$ and is denoted $\mathcal{H}_{n\ell,p}$. We write the projection operator onto $\mathcal{H}_{n\ell,p}$ by $P_{H_{n\ell}}$ so that we have $\mathcal{H}_{n\ell,p} = P_{H_{n\ell}}\mathcal{H}_{n\ell}$. The space orthogonal to the pure point spectral subspace $\mathcal{H}_{n\ell,p}$ is called continuous spectral subspace or scattering space of $H_{n\ell}$ and is denoted $\mathcal{H}_{n\ell,c}$, which is equal to $(I - P_{H_{n\ell}})\mathcal{H}_{n\ell}$. A state belonging to $\mathcal{H}_{n\ell,c}$ is called a scattering state.

When a state $f \in \mathcal{H}_{n\ell}$ is a bound state of $H_{n\ell}$ with eigenvalue $\lambda \in \mathbb{R}$, *i.e.* $H_{n\ell}f = \lambda f$ with the norm $\|f\| = 1$, the evolution $\exp(-itH_{n\ell})f$ of the state satisfies $\exp(-itH_{n\ell})f = \exp(-it\lambda)f$ so that the probability density of existence in configuration space is $|\exp(-itH_{n\ell})f(x)|^2 = |\exp(-it\lambda)f(x)|^2 = |f(x)|^2$ and this does not vary even when time parameter t changes so that it is a constant of motion and the state is considered to be stationary with respect to time. In this sense the system which starts with a bound state is regarded as timeless. Moreover that the probability density of the evolution is constant means that the system makes neither emission nor absorption of matter nor photon, so is regarded as an unobservable or invisible isolated system. (In our Hamiltonian (3), no photon is accommodated. However with the recent progress of quantum field theory in non-relativistic case (*e.g.* [10]), it will be possible to discuss photons in our formulation.) We call such a local system with the state $\exp(-itH_{n\ell})f$ that starts with a bound state $f \in \mathcal{H}_{n\ell,p}$ at the initial time $t = 0$ a timeless system or timeless local system. We remark that in a timeless system, the independent coordinates are the 6 components of configuration operator (x_1, x_2, x_3) and momentum operator (p_1, p_2, p_3) unlike the classical theory where the fundamental freedom of coordinates is 4 of time t and configuration (x_1, x_2, x_3).

When, on the other hand, a state $f \in \mathcal{H}_{n\ell}$ involves a scattering component $g \neq 0$ belonging to the continuous spectral subspace $\mathcal{H}_{n\ell,c}$, f is decomposed as an orthogonal sum $f = g + h$ with $h \in \mathcal{H}_{n\ell,p}$. In this case it is known that the following lemma holds.

As it requires complicated notation to describe the lemma in the case of general N-body system, we will in this paper be contented with considering merely the two body case $n = 1$. In this case the relative coordinate inside the two body system $(H_{1\ell}, \mathcal{H}_{1\ell})$ is $x = x_1 = X_2 - X_1 \in \mathbb{R}^3$, where X_j is the position of the j-th particle in $\mathbb{R}^3$ for $j = 1, 2$, and the reduced mass $\mu = \mu_1$ is given by $\mu = (m_2^{-1} + m_1^{-1})^{-1}$. Thus we have only to consider the Hamiltonian $H = H_{1\ell}$ of the form

$$H = H_0 + V(x), \quad H_0 = -\frac{1}{2\mu}\frac{\partial^2}{\partial x^2} = -\frac{1}{2\mu}\Delta \tag{4}$$

defined in $\mathcal{H} = \mathcal{H}_{1\ell} = L^2(\mathbb{R}^3)$. For the potential $V(x)$, we assume for instance the usual conditions as follows. $V(x)$ is decomposed as a sum $V(x) = V_S(x) + V_L(x)$ of two real-valued measurable functions $V_S(x)$, $V_L(x)$ of $x = (x_1, x_2, x_3) \in \mathbb{R}^3$ satisfying the following conditions. We recall that $\alpha = (\alpha_1, \alpha_2, \alpha_3)$ is a multi-index when $\alpha_j \geq 0$ is an integer, and $|\alpha| = \alpha_1 + \alpha_2 + \alpha_3$ is the length of α. We also use the notation $\langle x \rangle = \sqrt{1 + |x|^2}$ for $x \in \mathbb{R}^3$, $\partial_x = (\partial_{x_1}, \partial_{x_2}, \partial_{x_3}) = (\partial/\partial x_1, \partial/\partial x_2, \partial/\partial x_3)$, and $\partial_x^\alpha = \partial_{x_1}^{\alpha_1}\partial_{x_2}^{\alpha_2}\partial_{x_3}^{\alpha_3}$.

(*Short-range condition*) There exists a constant $0 < \delta < 1$ such that

$$\|\langle x \rangle^{1+\delta} V_S(x)(1 + H_0)^{-1}\| < \infty, \tag{5}$$

where $\|\cdot\|$ denotes the operator norm in $\mathcal{H} = L^2(\mathbb{R}^3)$.

(*Long-range condition*) Let $\delta \in (0, 1)$ be the same constant as in the above condition. For all multi-indices α there exists a constant $C_\alpha > 0$ such that for all $x \in \mathbb{R}^3$

$$|\partial_x^\alpha V_L(x)| \leq C_\alpha \langle x \rangle^{-|\alpha|-\delta}. \tag{6}$$

The short-range condition allows the local singularity for instance. It is the singularity like the one $|x|^{-2+\epsilon}$ with $\epsilon > 0$ when speaking about the singularity at the origin. This together with the long-range condition allows our potentials to include the Coulomb potential for example.

Then we have the following lemma (Theorem 3.2 in [11] for the general many body case, and Lemma 5.2 in [12] for the two body case). Noting that H does not have positive eigenvalues under the above conditions ([13]), we have only to consider the scattering state in the space $\mathcal{H}_c(a,b) = E_H([a,b])\mathcal{H}_c$ for $0 < a < b < \infty$, where E_H is the spectral measure for H.

Lemma 1.1. *For any $g \in \mathcal{H}_c(a,b)$ $(0 < a < b < \infty)$ with $\langle x \rangle^2 g \in \mathcal{H} = L^2(\mathbb{R}^3)$, there exists a sequence $t_k \to \pm\infty$ as $k \to \pm\infty$ such that for any $\phi \in C_0^\infty(\mathbb{R})$ and $R > 0$*

$$\|\chi_{\{x \in \mathbb{R}^3 | |x| < R\}} \exp(-it_k H) g\| \to 0, \tag{7}$$

$$\|(\phi(H) - \phi(H_0)) \exp(-it_k H) g\| \to 0, \tag{8}$$

$$\left\| \left(\frac{x}{t_k} - \frac{p}{\mu} \right) \exp(-it_k H) g \right\| \to 0 \tag{9}$$

as $k \to \pm\infty$, where $p = -i(\partial/\partial x_1, \partial/\partial x_2, \partial/\partial x_3)$ and χ_B denotes the characteristic function of a set B.

The relation (9) implies that on the evoluted state $\exp(-itH)g$ starting with a scattering state $g \in \mathcal{H}_c$, the asymptotic relation

$$\frac{x}{t_k} \sim \frac{p}{\mu} \quad (t_k \to \pm\infty) \tag{10}$$

holds. We note that p/μ is the relative velocity of the two particles. The relation (10) then means that the classical velocity x/t_k agrees with the quantum mechanical velocity p/μ asymptotically as $t_k \to \pm\infty$ in the phase space $\mathbb{R}^3 \times \mathbb{R}^3$ of (x,p) on the evoluted state $\exp(-it_k H)g$ when g is a scattering state of H. Namely micro-locally, the quantum mechanical picture of motion agrees asymptotically with the classical picture of motion when the system starts with a scattering state. This as well implies that the two body system $(H, \mathcal{H})$ is internally alive in the sense that the probability density $|\exp(-itH)g(x)|^2/\|g\|^2$ varies along with the change of the parameter t. These results extend to general N-body case as shown in Theorem 3.2 in [11]. This justifies the definition that the local time t of the general local system $(H_{n\ell}, \mathcal{H}_{n\ell})$ is given by the parameter t on the exponent of the evolution $\exp(-itH_{n\ell})f$ of the system when the system starts with a state f involving a component $g \neq 0$ belonging to the scattering space $\mathcal{H}_{n\ell,c}$ of $H_{n\ell}$. Further lemma 1.1 implies that one can take the 4 components of independent coordinates of local time t and the configuration coordinates (x_1, x_2, x_3) as the fundamental free coordinates which describe the internal motion of the local system as well as implies that the system is considered observable as it scatters and emits the parts to the outside of the system. (We refer the reader to [11] for a detailed argument on this subject.) We denote this time t by $t_{(H_{n\ell}, \mathcal{H}_{n\ell})}$ indicating the local system under consideration. We call such visible or observable local systems just local systems when no confusion arises.

On the contrary as we have seen, when the system starts with a bound state f of the Hamiltonian $H_{n\ell}$ with $H_{n\ell}f = \lambda f$, $f \neq 0$ and $\lambda \in \mathbb{R}$, the system is regarded to have no time coordinate and is called a timeless system. Its fundamental coordinates are the 6 components of the position (x_1, x_2, x_3) and momentum (p_1, p_2, p_3), and the timeless system is considered unobservable or invisible.

2 Local systems as classical particles

There are zillions of observable local systems in our universe. Each of those local systems is a quantum mechanical system and is regarded as a classical particle as usual with its center of mass being identified with a classical particle. In fact as the center of mass and the internal relative coordinates of a local system are mutually independent, we could remove the center of mass of each local system when we consider the internal motion, as stated above. Utilizing this mutual independence between the internal coordinates and the center of mass of a local system, we can assume general theory of relativity among the centers of mass of those local systems and can prove that the postulates of general relativity posed on the centers of mass of local systems are consistent with the postulates of quantum mechanics assumed on the internal motion of each local system. Namely we have proved the following theorem in Theorem

2 of [5].

To state the theorem we first note that, with utilizing the definition of the local time $t = t_{(H_{n\ell},\mathcal{H}_{n\ell})}$ of the system $(H_{n\ell},\mathcal{H}_{n\ell})$ which has nonzero scattering space, we can define the local space-time $(t,x) = (t_{(H_{n\ell},\mathcal{H}_{n\ell})},\ x_{(H_{n\ell},\mathcal{H}_{n\ell})}) \in \mathbb{R}^4$ of the local system $(H_{n\ell},\mathcal{H}_{n\ell})$ such that the center of mass of the local system $(H_{n\ell},\mathcal{H}_{n\ell})$ is at the origin $x = 0$ of the space coordinates $x = (x_1, x_2, x_3)$ of $\mathbb{R}^3$.

We are now in a position to state our postulates of relativity on the centers of mass of local systems.

General principle of relativity. The laws of physics which govern the relative motion of the centers of mass of the observed local systems are expressed by the classical equations which are covariant under the change of observer's coordinate systems of $\mathbb{R}^4$ from one observer's coordinates $(t,x) = (t_{(H_{mk},\mathcal{H}_{mk})}, x_{(H_{mk},\mathcal{H}_{mk})})$ to another observer's coordinates $(t,x) = (t_{(H_{n\ell},\mathcal{H}_{n\ell})}$, $x_{(H_{n\ell},\mathcal{H}_{n\ell})})$ for any pairs (m,k), (n,ℓ).

It is included in this postulate that one can observe the positions of the centers of mass of other systems in his coordinate system (t,x). The relative velocities of the observed systems are then defined as quotients of the relative positions of those systems and the local time t of the observer's system.

Principle of equivalence. The coordinate system $(t_{(H_{n\ell},\mathcal{H}_{n\ell})}, x_{(H_{n\ell},\mathcal{H}_{n\ell})})$ associated with the local system $(H_{n\ell},\mathcal{H}_{n\ell})$ is the local Lorentz system of coordinates. Namely, the gravitational tensor $g_{\mu\nu}$ for the center of mass of the local system $(H_{n\ell},\mathcal{H}_{n\ell})$, observed in these coordinates $(t_{(H_{n\ell},\mathcal{H}_{n\ell})}, x_{(H_{n\ell},\mathcal{H}_{n\ell})})$, are equal to $\eta_{\mu\nu}$. Here $\eta_{\mu\nu} = 0$ $(\mu \neq \nu)$, $= 1$ $(\mu = \nu = 1,2,3)$, and $= -1$ $(\mu = \nu = 0)$.

The principle of equivalence together with the general principle of relativity implies that for the coordinate system $(t_{(H_{n\ell},\mathcal{H}_{n\ell})}, x_{(H_{n\ell},\mathcal{H}_{n\ell})})$ associated with the local system $(H_{n\ell},\mathcal{H}_{n\ell})$, the principle of constancy of the velocity of light holds in the following sense: The light radiated from another system $(H_{mk},\mathcal{H}_{mk})$ moving with a steady velocity relative to an observer's system $(H_{n\ell},\mathcal{H}_{n\ell})$ propagates through the flat region where $g_{\mu\nu} = \eta_{\mu\nu}$ at a constant velocity c, independently of the velocity of the system $(H_{mk},\mathcal{H}_{mk})$ relative to the observer's system $(H_{n\ell},\mathcal{H}_{n\ell})$.

In particular, those principles imply the Lorentz transformation which connects the two inertial frames of reference which move each other with a constant velocity.

Under these conditions we have proved the following consistency theorem between quantum mechanics and general theory of relativity in Theorem 2 of [5].

Theorem 2.1. *General principle of relativity and the principle of equivalence postulated on the relative motion between the centers of mass of various local systems are consistent with the postulate of quantum mechanics posed on the internal motion of each local system* $(H_{n\ell}, \mathcal{H}_{n\ell})$*, i.e. consistent with the postulate that the internal relative motion of particles inside the system* $(H_{n\ell}, \mathcal{H}_{n\ell})$ *is described by the Hamiltonian* $H_{n\ell}$ *defined by (3).*

For the proof we refer the reader to [5].

As a special case, we have discussed in [4] the case where the velocity of an observable local system is constant relative to an observer with some postulates on the relation between the internal velocity of the system and the external velocity of its center of mass with respect to the observer. With those postulates we have deduced the same consequence as the special theory of relativity and, in particular the Lorentz transformation, showing that the quantum mechanical local clock is equivalent to the classical relativistic clock.

On the other hand, with regards to the invisible timeless local systems, those principles of general relativity do not apply as those systems have no time coordinate. In particular, the fact that time is not defined in such a system yields that the Lorentz transformation does not apply to timeless systems even if the system moves with a constant velocity relative to an observer. Therefore we can assume in a consistent manner with the theory of relativity that the mass of such a timeless system is constant, independently of its velocity relative to the observer. Namely we can assume the following on timeless local systems consistently with the principles of general relativity.

Principle of constancy of mass of timeless systems. The mass of the timeless unobservable system $\exp(-itH_{n\ell})f$ which starts with a bound state $f \in \mathcal{H}_{n\ell,p}$ with $H_{n\ell}f = \lambda f$, $f \neq 0$, $\lambda \in \mathbb{R}$ at the initial time $t = 0$ is a constant, independently of its velocity relative to the observer.

We remark that there might be other possible consistent assumptions on those timeless systems. The problem which assumption is appropriate will be decided by the future investigation both from experimental and theoretical perspectives. Considering the nature of the timeless system that it has no time, it would be a natural assumption that the mass of such a timeless system does not change and is independent of its velocity relative to the observer, however.

3 Superluminal phenomena, dark matter and Big Bang

As we have seen in the previous section, if a state $\exp(-itH_{n\ell})f$ starts at the initial time $t = 0$ with an eigenstate $f \in \mathcal{H}_{n\ell,p}$ of the Hamiltonian $H_{n\ell}$, the system is regarded to have no time coordinate as well as it is unobservable or invisible. We called such a system a timeless system or timeless local system.

A feature of timeless local systems is that they have no coordinate system which includes time coordinate. Therefore the principles of general theory of relativity postulated in the previous section do not apply to such a timeless system. Hence it is consistent with the theories of relativity to assume the principle of constancy of mass of timeless systems stated at the end of the previous section. Then the mass of the timeless system will not increase nor change until it becomes observable by some causes even if it moves with a speed close to that of light relative to the observer. Therefore such a timeless system $\exp(-itH_{n\ell})f$ with $H_{n\ell}f = \lambda f$, $f \neq 0$, $\lambda \in \mathbb{R}$ could travel with an arbitrary speed, *e.g.* with a superluminal speed, relative to the observer accordingly to the force applied to it at the initial time $t = 0$, until the system turns into a state involving scattering component at a time s by some causes like colliding with other local system $(H_{mk}, \mathcal{H}_{mk})$ so that the state $\exp(-isH_{n\ell})f$ becomes a state $\exp(-isH_{n\ell})h$ involving a nonvanishing scattering component $\exp(-isH_{n\ell})g$ ($g \in \mathcal{H}_{n\ell,c} = (I - P_{H_{n\ell}})\mathcal{H}_{n\ell}$) with respect to the Hamiltonian $H_{n\ell}$ when considered in the combined system $(H_{pj}, \mathcal{H}_{pj})$ of $(H_{n\ell}, \mathcal{H}_{n\ell})$ and $(H_{mk}, \mathcal{H}_{mk})$. At this moment s, the state $\exp(-isH_{n\ell})h$ involves a scattering state so that it has its local time. Consequently it becomes observable and appears in the real world at the position of the collision with the system $(H_{mk}, \mathcal{H}_{mk})$. An important remark at this point is that this superluminal travel does not violate the principle of causality. In the usual theory of relativity, it is assumed that in a superluminal travel, the direction of the traveler's time is reversed and he will be back to the past at the arrival. But in our timeless system, the time inside it does not change so that it arrives at the destination at the same time as its departure, but in the observer's framework a nonzero positive time has passed. Thus the system arrives at the destination at a future time in the observer's time coordinate.

Utilizing these considerations, we would be able to construct a warp drive system for example under the before-metioned principle of constancy of mass of timeless systems. Given a local system $(H_{n\ell}, \mathcal{H}_{n\ell})$, if one can make it a bound state with respect to the local Hamiltonian $H_{n\ell}$, he can send that timeless system with a superluminal velocity relative to the observer to a target by applying an appropriate force at the initial time. If it remains the bound state it is not observable and it is the same that it does not exist. Nonetheless if one can wake up the local system at the destination to turn a system to a state which possesses a component of scattering state which belongs to the continuous spectral subspace $\mathcal{H}_{n\ell,c} = (I - P_{H_{n\ell}})\mathcal{H}_{n\ell}$ of $H_{n\ell}$, then the system obtains time coordinate and becomes observable. A timeless system $(H_{n\ell}, \mathcal{H}_{n\ell})$ in a bound state wakes up to obtain time coordinate, when it collides with another system $(H_{mk}, \mathcal{H}_{mk})$ for instance as stated above. At the collision the bound state would be broken into several subsystems in the combined system $(H_{pj}, \mathcal{H}_{pj})$ of $(H_{n\ell}, \mathcal{H}_{n\ell})$ and $(H_{mk}, \mathcal{H}_{mk})$ so that the system $(H_{n\ell}, \mathcal{H}_{n\ell})$ becomes involving a scattering component with respect to the system's Hamiltonian $H_{n\ell}$. If we can send out a system in a bound state toward some object with applying a sufficient amount of energy, and if the collision with the target takes place in an appropriate manner, the system will appear in the real world at the target as a state involving scattering component and time coordinate. The mean velocity from the start till the appearance at the target will be superluminal.

We note that the local time of the timeless system is frozen just as in the biological hypothermic hibernation of living things. In this sense we may as well call a timeless system as a physically hibernated system or simply a frozen system. The travel in the timeless system is therefore the one in which the system is frozen as a local system. It should be remarked that this does not mean that sublocal systems of the frozen system are also frozen however. On the contrary the subsystems can be alive just as the local systems inside the total stationary timeless universe ϕ considered in [5] can be alive and each can have the local time of its own ([11], [14], [15]).

We remark that such a frozen system would be abundant in nature and countless such systems are around us with being unnoticed. Some of them would appear in the real world by some chance. For example, the creation and annihilation of elementary particles would be one of such phenomena. What we can notice of them is only their appearance usually, so that we rarely know that they had travelled faster-than-light.

Such timeless, frozen systems would be the one that has been called "dark matter" in the sense that each of those systems is unobservable as remarked above but has mass of its own. Until those systems appear in the real world by some causes like collision with other systems they are hidden and unseen because they are eigenstates of some Hamiltonians. Those systems have mass however, which will explain the invisible dark matter.

The fact that timeless systems are not observable will give an explanation of the so-called Big Bang phenomenon as a collision of plural timeless local systems whether they had travelled faster-than-light or slower-than-light until the collision. If we think the Big Bang as such a collision of timeless unobservable systems, it will give an explanation consistent with the conservation law of matter or energy of the observed fact that no emission of photon nor other matter before some 13.7 billion years ago has been observed.

4 Conclusion

We have reviewed the notion of local system and local time, and distinguished the two cases of local systems. One is the timeless and unobservable local system, and another is the observable local system with the associated local time. The former timeless local system is a bound state or an eigenstate of the local Hamiltonian of the system, and is regarded timeless because it does not change according to its time variation. Further as an eigenstate of the local Hamiltonian, the timeless system emits nothing outside, so is unobservable.

The nature that the timeless system does not have time defined yields a possibility, consistently with the theory of relativity, that its mass might be constant regardless of its velocity relative to the observer, so implies a possibility that such a system could travel with a superluminal speed according to the energy applied to it at the initial time.

Another nature that the timeless system is not observable suggests an explanation of dark matter as timeless systems as each of those systems has mass of its own although they are unobservable. The nature that those timeless systems are unobservable as well would give an explanation of Big Bang as a collision of plural unobservable timeless systems some billions years ago, which is consistent with the conservation law of matter or energy and the fact that no emission of photon nor matter has been observed before about 13.7 billion years ago.

References

[1] D. V. Ahluwalia, *On a new non-geometric element in gravity*, Gen. Rel. Grav. **29, No. 12** (1997), 1491-1501.

[2] D. V. Ahluwalia and C. Burgard, *Interplay of gravitation and linear superposition of different mass eigenvalues*, Phys. Rev. D **57** (1998), 4724-4727.

[3] G. Z. Adunas, E. Rodriguez-Milla, and D. V. Ahluwalia, *Probing quantum aspects of gravity*, Phys. Lett. B **485** (2000), 215-223.

[4] H. Kitada, *Quantum Mechanical Clock and Classical Relativistic Clock*, (gr-qc/0102057) (2001).

[5] H. Kitada, *Theory of local times*, Il Nuovo Cimento, **109** B (1994), No. 3, 281-302.

[6] H. Kitada, *Theory of local times II. Another formulation and examples* (http://xxx.lanl.gov/abs/gr-qc/9403007) (1994).

[7] H. Kitada and L. Fletcher, *Local time and the unification of physics Part I. Local time*, Apeiron, **3** (1996), No. 2, 38-45.

[8] H. Kitada, *Quantum mechanics and relativity — Their unification by local time —* in "Spectral and Scattering Theory," ed. A. G. Ramm, Plenum Press, New York, 1998, pp.39-66.

[9] H. Kitada, *Local Time and the Unification of Physics Part II. Local System*, (gr-qc/0110066) (2001).

[10] I. M. Sigal, *Renormalization group and problem of radiation*, arXiv:1110.3841 (2011).

[11] H. Kitada, Quantum Mechanics, Lectures in Mathematical Sciences, vol. 23, The University of Tokyo, 2005, x + 168 pp. ISSN 0919-8180, ISBN 1-000-01896-2. (http://arxiv.org/abs/quant-ph/0410061)

[12] H. Kitada, *Scattering theory for the fractional power of negative Laplacian*, J. Abstr. Differ. Equ. Appl., **1** (2010), No. 1, 1-26 (http://math-res-pub.org/images/stories/abstr11.pdf).

[13] R. Froese and I. Herbst, *Exponential bounds and absence of positive eigenvalues for N-body Schrödinger operators*, Commun. Math. Phys. **87** (1982), 429-447.

[14] H. Kitada and L. Fletcher, *Comments on the Problem of Time — A response to "A Possible Solution to the Problem of Time in Quantum Cosmology" by Stuart Kauffman and Lee Smolin*, (gr-qc/9708055) (1997).

[15] H. Kitada, *A possible solution for the non-existence of time*, (gr-qc/9910081) (1999).