Theory of Local Times
II. Another formulation and examples

By

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Abstract. The model of a stationary universe and the notion of local times presented in [10] are reviewed with some alternative formulation of the consistent unification of the Riemannian and Euclidean geometries of general relativity and quantum mechanics. The method of unification adopted in the present paper is by constructing a vector bundle $X \times R^6$ or $X \times R^4$ with X being the observer's reference frame and R^6 or R^4 being the unobservable inner space(-time) within each observer's local system. Some applications of our theory to two concrete examples of human size and of cosmological size are discussed, as well as the uncertainty of time in our context is calculated.

1 Introduction

As stated in Section 2 of [10], it seems that there are fundamental difficulties in the presently existing theories trying to unify the relativity and quantum mechanics.

We review quickly and briefly some fundamental difficulties of these theories that try to unify the quantum and relativity theories.

The difficulties of the existing theories:

1) The quantum field theories—unification of special relativity and quantum theory

In the trials in this direction, there is the difficulty of the divergence, or the problem of renormalization. According to the Euclidean method in the axiomatic quantum field theory that is a mathematically rigorous investigation in this direction, the following is known. We denote by ν the spacetime dimension:

a) For $\nu \geq 5$, there are no non-trivial models.

b) For $\nu = 2, 3$, there are non-trivial models.

c) For $\nu = 4$, there are no non-trivial models, if one use the lattice approximation with some additional assumptions on the renormalizability of the theories.

For the details, see, e.g., Streater's paper in [3], Fröhlich [8].

2) The quantum gravity—unification of general relativity and quantum theory

In the quantum mechanics, time occupies a special position as in the Newtonian mechanics. The Newtonian time is a concept which is incompatible with diffeomorphism-invariance. Here is a central problem. Namely if the general relativistic quantum mechanics is completed, then the (proper) time should be defined as an invariant quantity with respect to the diffeomorphisms group of spacetime. But in the quantum theories, time remains an absolute notion in the Newtonian sense. There are various trials in treating this difficulty.

Among the trials where the problem is regarded as the quantization of general relativity, there are two directions of trials. One is to identify time before quantization, and another is to identify time after quantization.

There are also trials without assuming the notion of time.

For details, see, e.g., Isham [9]. See also Ashtekar and Stachel [2].

In [10], we presented a formulation, where the notion of local times eliminates several fundamental difficulties mentioned above.

The purpose of the present part II is to give some simpler exposition of the consistent unification of quantum and relativity theories with clarifying the role of the observer's systems, and their relation with the inner spacetime within each local system. In short, the unification in this part II is done by orthogonalizing the observer's coordinates (= Riemannian spacetime) to the Euclidean spaces inside each local system. This presentation is a realization of the remark stated in Section 6 of [10], and will be done in Sections 2–4 below with some repetitions of the axioms and definitions stated in [10].

As a concrete exposition of our theory, we will give in Section 5 an explanation of the experiment of the interference of one neutron in the uniform gravitational field, done by Collela et al. [4]. We will also explain the Hubble's redshift in Section 6, which will be done in quite the same way as in the general relativity theory. In the final section 7, we will calculate the order of the uncertainty of time which is predicted by our theory.

2 Local times

Fundamental 3 axioms:

Let $\mathcal{H}$ be a separable Hilbert space, and set

$$\mathcal{U} = \{\phi\} = \bigoplus_{n=0}^{\infty} \left(\bigoplus_{\ell=0}^{\infty} \mathcal{H}^n \right) \qquad (\mathcal{H}^n = \underbrace{\mathcal{H} \otimes \cdots \otimes \mathcal{H}}_{n \text{ factors}}). \tag{2.1}$$

$\mathcal{U}$ is called a (separable) Hilbert space of possible universes.

Let $\mathcal{O} = \{A\}$ be the totality of the selfadjoint operators A in $\mathcal{U}$ of the form $A\phi = (A_{n\ell}\phi_{n\ell})$ for $\phi = (\phi_{n\ell}) \in \mathcal{U}$ in the domain of the operator A.

Axiom 1. There exist a selfadjoint operator $H \in \mathcal{O}$ in $\mathcal{U}$ such that for some $\phi \in \mathcal{U} - \{0\}$ and $\lambda \in R^1$

$$H\phi = \lambda\phi \tag{2.2}$$

in the following sense: There exists an infinite matrix $(\lambda_{n\ell})$ of real numbers such that $H_{n\ell}\phi_{n\ell} = \lambda_{n\ell}\phi_{n\ell}$ for each $n \geq 1$, $\ell \geq 0$ and $\lambda_{n\ell_n} \to \lambda$ as $n \to \infty$ along any ℓ_n such that $F_n^{\ell_n} \subset F_{n+1}^{\ell_{n+1}}$. Here F_n is a finite subset of $\mathbf{N} = \{1, 2, \cdots\}$ with $\sharp(F_n) =$ (the number of elements in F_n) $= n$ and $\{F_n^\ell\}_{\ell=0}^\infty$ is the totality of such F_n.

H is an infinite matrix $(H_{n\ell})$ of selfadjoint operators $H_{n\ell}$ in $\mathcal{H}^n$. Axiom 1 asserts that this matrix converges in the sense of (2.2).

Axiom 2. Let $n \geq 1$ and F_{n+1} be a finite subset of $\mathbf{N} = \{1, 2, \cdots\}$ with $\sharp(F_{n+1}) = n+1$. Then for any $j \in F_{n+1}$, there exist selfadjoint operators $X_j = (X_{j1}, X_{j2}, X_{j3})$, $P_j = (P_{j1}, P_{j2}, P_{j3})$ in $\mathcal{H}^n$ and constants $m_j > 0$ such that

$$[X_{j\ell}, X_{km}] = 0, \qquad [P_{j\ell}, P_{km}] = 0, \qquad [X_{j\ell}, P_{km}] = i\delta_{jk}\delta_{\ell m}, \tag{2.3}$$

$$\sum_{j \in F_{n+1}} m_j X_j = 0, \qquad \sum_{j \in F_{n+1}} P_j = 0. \tag{2.4}$$

By the Stone-von Neumann theorem Axiom 2 also specifies the space dimension (see Abraham-Marsden [1], p.452). We identify $\mathcal{H}^n$ with $L^2(R^{3n})$ in the following.

What we want to mean by $H_{n\ell}$ $(n, \ell \geq 0)$ in Axiom 1 is the $N = n+1$ body Hamiltonian in the usual quantum mechanics. For the local Hamiltonian $H_{n\ell}$ we thus make the following postulate.

Axiom 3. Let $n \geq 0$ and F_N $(N = n+1)$ be a finite subset of $\mathbf{N} = \{1, 2, \cdots\}$ with $\sharp(F_N) = N$. Let $\{F_N^\ell\}_{\ell=0}^\infty$ be the totality of such F_N. Then the Hamiltonians $H_{n\ell}$ $(\ell \geq 0)$ are of the form

$$H_{n\ell} = H_{n\ell 0} + V_{n\ell}, \qquad V_{n\ell} = \sum_{\alpha=(i,j), 1 \leq i < j < \infty, i,j \in F_N^\ell} V_\alpha(x_\alpha) \tag{2.5}$$

on $C_0^\infty(R^{3n})$, where $x_\alpha = x_i - x_j$ with x_i being the position vector of the i-th particle, and $V_\alpha(x_\alpha)$ is a real-valued measurable function of $x_\alpha \in R^3$ which is $H_{n\ell 0}$-bounded with $H_{n\ell 0}$-bound of $V_{n\ell}$ less than 1. $H_{n\ell 0} = H_{(N-1)\ell 0}$ is the free Hamiltonian of the N-particle system. The concrete form is expressed as in [11], (1.4), if one use clustered Jacobi coordinates.

This axiom implies that $H_{n\ell} = H_{(N-1)\ell}$ is uniquely extended to a selfadjoint operator in $\mathcal{H}^n = \mathcal{H}^{N-1} = L^2(R^{3(N-1)})$ by the Kato-Rellich theorem.

A theorem in many-body scattering theory:

For the N-body Hamiltonian $H_{N-1} = H_{n\ell}$ $(N = n+1)$ the following Theorem 1 is known [6] to hold under suitable assumptions (e.g., Assumptions 1 and 2 in [11]).

We here follow the notation and conventions in [11] for the N-body quantum systems. In particular $H_b = H_{(N-1)b} = H_{N-1} - I_b = H^b_{n\ell} + T_{n\ell b} = H^b + T_b$ is the truncated Hamiltonian for the cluster decomposition $1 \le |b| \le N$, and P_b^M is the M-dimensional partial projection of the eigenprojection $P_b = P_{H^b}$ associated with the subsystem H^b, i.e., P_b is the orthogonal projection in $\mathcal{H}^b = L^2(R^{3(N-|b|)})$ onto the eigenspace of H^b. q_b is the velocity conjugate to the intercluster coordinates x_b. We define for a k-dimensional multi-index $M = (M_1, \cdots, M_k)$, $M_j \ge 1$,

$$\hat{P}_k^M = \left(I - \sum_{|b|=k} P_b^{M_k}\right) \cdots \left(I - \sum_{|d|=2} P_d^{M_2}\right)(I - P^{M_1}), \tag{2.6}$$
$$k = 1, \cdots, N-1,$$

where $P^{M_1} = P_a^{M_1}$ with $|a| = 1$, and for a $|b|$-dimensional multi-index $M_b = (M_1, \cdots, M_{|b|-1}, M_{|b|}) = (\hat{M}_b, M_{|b|})$

$$\tilde{P}_b^{M_b} = P_b^{M_{|b|}} \hat{P}_{|b|-1}^{\hat{M}_b}, \qquad 2 \le |b| \le N. \tag{2.7}$$

It is clear that

$$\sum_{2 \le |b| \le N} \tilde{P}_b^{M_b} = I - P^{M_1}, \tag{2.8}$$

provided that the component M_k of M_b depends only on the number k but not on b. In the following we use such M_b's only. Under these circumstances, the following is known to hold.

Theorem 1 ([6] Enss). Let Assumptions 1 and 2 in [11] be satisfied. Let $f \in \mathcal{H}^{N-1}$. Then there are a sequence $t_m \to \pm\infty$ (as $m \to \pm\infty$) and a sequence M_b^m of multi-indices whose components all tend to ∞ as $m \to \pm\infty$ such that for all cluster decompositions b, $2 \le |b| \le N$, $\psi \in C_0^\infty(R^1)$, and $\varphi \in C_0^\infty(R^{3(|b|-1)})$

$$\left\| \frac{|x^b|^2}{t_m^2} \tilde{P}_b^{M_b^m} e^{-it_m H_{N-1}} f \right\| \to 0, \tag{2.9}$$

$$\|\{\psi(H_{N-1}) - \psi(H_b)\} \tilde{P}_b^{M_b^m} e^{-it_m H_{N-1}} f\| \to 0, \tag{2.10}$$

$$\|\{\varphi(x_b/t_m) - \varphi(q_b)\}\tilde{P}_b^{M_b^m} e^{-it_m H_{N-1}} f\| \to 0 \tag{2.11}$$

as $m \to \pm\infty$.

Definition of local times:

Definition 1.

- Let $\phi = (\phi_{n\ell})$ with $\phi_{n\ell} = \phi_{n\ell}(x_1, \cdots, x_n) \in L^2(R^{3n})$ be the universe in Axiom 1.
- We define $\mathcal{H}_{n\ell}$ as the sub-Hilbert space of $\mathcal{H}^n$ generated by the functions $\phi_{mk}(x^{(\ell)}, y)$ of $x^{(\ell)} \in R^{3n}$ with regarding $y \in R^{3(m-n)}$ as a parameter, where $m \geq n$, $F^\ell_{n+1} \subset F^k_{m+1}$, and $x^{(\ell)}$ is the (relative) coordinates of $n+1$ particles in F^ℓ_{n+1}.
- $\mathcal{H}_{n\ell}$ is called a **local universe** of ϕ.
- $\mathcal{H}_{n\ell}$ is said to be non-trivial if $(I - P_{H_{n\ell}})\mathcal{H}_{n\ell} \neq \{0\}$.

The total universe ϕ is a single element in $\mathcal{U}$. The local universe $\mathcal{H}_{n\ell}$ may be richer and may have elements more than one. This is because we consider the subsystems of the universe consisting of a finite number of particles. These subsystems receive the influence from the other particles of infinite number outside the subsystems, and may vary to constitute a non-trivial subspace $\mathcal{H}_{n\ell}$.

Definition 2.

- The restriction of H to $\mathcal{H}_{n\ell}$ is also denoted by the same notation $H_{n\ell}$ as the (n,ℓ)-th component of H.
- We call the pair $(H_{n\ell}, \mathcal{H}_{n\ell})$ a local system.
- The unitary group $e^{-itH_{n\ell}}$ $(t \in R^1)$ on $\mathcal{H}_{n\ell}$ is called the **proper clock** of the local system $(H_{n\ell}, \mathcal{H}_{n\ell})$, if $\mathcal{H}_{n\ell}$ is non-trivial: $(I - P_{H_{n\ell}})\mathcal{H}_{n\ell} \neq \{0\}$.
- Note that the clock is defined only for $N = n+1 \geq 2$, since $H_{0\ell} = 0$.
- The universe ϕ is called **rich** if $\mathcal{H}_{n\ell}$ equals $\mathcal{H}^n = L^2(R^{3n})$ for all $n \geq 1$, $\ell \geq 0$. For a rich universe ϕ, $H_{n\ell}$ equals the (n,ℓ)-th component of H.

The formula (2.11) indicates that t_m is asymptotically equal to $\pm|x_b|/|q_b|$ as $m \to \pm\infty$, independently of the choice of cluster decompositions b. This is precisely the actual procedure of measuring the time t_m in the mechanics. The implication of this theorem is therefore interpreted as follows: If one 'measures' the time of a state $f \in (I - P_{H_{(N-1)\ell}})\mathcal{H}_{(N-1)\ell} - \{0\}$ in the local system $(H_{(N-1)\ell}, \mathcal{H}_{(N-1)\ell})$ by the associated proper clock $e^{-itH_{(N-1)\ell}}f$, namely if one measures the quotient $\pm|x_b|/|q_b|$ of the scattered particles which are regarded as moving almost in a steady velocity, then that time is asymptotically equal to the parameter t_m in the exponent of $e^{-it_m H_{(N-1)\ell}}f$ as $m \to \pm\infty$. In this sense t_m is interpreted as the

quantum mechanical proper time of the local system $(H_{n\ell}, \mathcal{H}_{n\ell}) = (H_{(N-1)\ell}, \mathcal{H}_{(N-1)\ell})$, if $(I - P_{H_{(N-1)\ell}})\mathcal{H}_{(N-1)\ell} \neq \{0\}$.

Definition 3.

- The parameter t in the exponent of the proper clock $e^{-itH_{n\ell}} = e^{-itH_{(N-1)\ell}}$ of the local system $(H_{n\ell}, \mathcal{H}_{n\ell})$ is called the (quantum mechanical) **proper time** or **local time** of the local system $(H_{n\ell}, \mathcal{H}_{n\ell})$, if $(I - P_{H_{n\ell}})\mathcal{H}_{n\ell} \neq \{0\}$.
- This time t is denoted by $t_{(H_{n\ell}, \mathcal{H}_{n\ell})}$ indicating the local system under consideration.

This definition is the one reverse to the usual definition of the motion or dynamics of the N-body quantum systems, where the time t is given *a priori* and the motion of the particles is defined by $e^{-itH_{(N-1)\ell}}f$ for a given initial state f of the system.

Time is thus defined only for the local systems $(H_{n\ell}, \mathcal{H}_{n\ell})$ and is determined by the associated proper clock $e^{-itH_{n\ell}}$. Therefore there are infinitely many times $t = t_{(H_{n\ell}, \mathcal{H}_{n\ell})}$ each of which is proper to the local system $(H_{n\ell}, \mathcal{H}_{n\ell})$. In this sense time is a local notion. There is no time for the total universe ϕ in Axiom 1, which is a (stationary) bound state for the total Hamiltonian H.

Uncertainty relation among time, position and momentum:

This local time is an approximate one in a double sense.

- First t_m is only *asymptotically* equal to $\pm|x_b|/|q_b|$ as $m \to \pm\infty$.

  This fact explains the so-called principle of uncertainty in our context. In the usual explanation, the position x_b and the velocity q_b or the momentum p_b cannot be determined in equal accuracy. According to our theory, this is rephrased as follows: The time t cannot be determined accurately, even if x_b and q_b could be determined precisely. It is only determined in some *mean* sense as in (2.11). From the usual uncertainty relation $\Delta x \cdot \Delta p \geq \hbar/2$ and $p = mq$ follows $\Delta x \cdot \Delta q \geq \hbar/(2m)$, which indicates that the uncertainty of time is proportional to m^{-1}. This explanation resolves the difficulty of the uncertainty between time and energy when one considers the time as an operator. (We will give a concrete explanation in Section 7.)

- Second the local Hamiltonian $H_{n\ell}$ is not the total Hamiltonian H. Or rather, the time arises from this approximation of H by $H_{n\ell}$.

  This approximation may make $\mathcal{H}_{n\ell}$ non-trivial, and the clock $e^{-itH_{n\ell}}$ can be defined as in Definition 2 owing to $(I - P_{H_{n\ell}})\mathcal{H}_{n\ell} \neq \{0\}$.

  On the contrary the total universe ϕ has no associated clock and time, since $(I-P_H)\phi = 0$.

Mutual independency of local systems:

Our theory of local times further implies in particular that local systems $(H_{n\ell}, \mathcal{H}_{n\ell})$ are *mutually independent*.

Indication of Proof: As a typical example, let us consider the case $F_{N'}^{\ell'} \subset F_N^{\ell}$ with $N' < N$. In this case, $H_{(N'-1)\ell'}$ is a subsystem Hamiltonian of $H_{(N-1)\ell}$. However the correspondent times $t_{N'\ell'}$ and $t_{N\ell}$ are measured mutually independently as in Theorem 1-(2.11).

The problem that may arise in this case is with the common variable $x^{(\ell')}$. But as $\mathcal{H}_{(N'-1)\ell'}$ and $\mathcal{H}_{(N-1)\ell}$ indicate, these spaces have different (ℓ', N') and (ℓ, N). Thus by (2.1) before Definition 1, these spaces $\mathcal{H}_{(N'-1)\ell'}$ and $\mathcal{H}_{(N-1)\ell}$ are the subspaces of the ℓ'-th $\mathcal{H}^{(N'-1)}$ and ℓ-th $\mathcal{H}^{(N-1)}$ of the formula (2.1), hence are mutually independent Hilbert spaces. This implies that the L^2-representations of these spaces are also mutually independent. Therefore the correspondent clocks and local times are also mutually independent.

Interpretation of usual quantum mechanics:

We have defined the (local) time $t = t_{(H_{n\ell}, \mathcal{H}_{n\ell})}$ for each local system $(H_{n\ell}, \mathcal{H}_{n\ell})$. This time t satisfies Theorem 1-(2.11). If one regards the time t as a *given* quantity, this fact is interpreted as follows: In each local system $(H_{n\ell}, \mathcal{H}_{n\ell})$, the physics follows the quantum mechanics, i.e., follows the Schrödinger equation.

Toward the relativity:

Our definition of times is consistent with the theory of (general) relativity of Einstein. Our (quantum mechanical) proper time of the local system $(H_{n\ell}, \mathcal{H}_{n\ell})$ can be regarded as the quantum mechanical correspondent to the classical proper time in the theory of relativity.

3 Relativity

For the relative motions of the *centers* of mass of local systems, we postulate the principle of (general) relativity and the principle of equivalence as in Einstein [7].

Fundamental assumptions on 'observable' and 'unobservable':

What should be stated first on our introduction of relativity is that we make the following fundamental assumptions:

- **Only** the relative classical motions of the **centers** of mass of local systems are **observable** in our theory.
- The **internal** quantum mechanical motion within each local system is independent of **observation**, at the present stage of our theory. In this sense, the internal quantum mechanical motion within a local system is **unobservable**.
- We postulate Axiom 6 in the next section 4, which gives a principle of **deduction** of the **internal** quantum mechanical motion within each local system from classical

observations of its **sub** local systems, through certain relativistic considerations.

Identification of local time with the relativistic proper time:

Fix one observer's local system $L_O = (H_{n\ell}, \mathcal{H}_{n\ell})$. Then the classical world observable by L_O is observed within the 4-dimensional Riemann space X, whose time is assumed to coincide with the local time t of that local system L_O at the center of mass of L_O, and whose origin of the space coordinates is assumed to be equal to the L_O's center of mass.

The unobservable inner space associated with the local system L_O is the Euclidean space R^6 consisting of the points (x, p), where x is the configuration and p is the momentum conjugate to x. The local time t is defined as the parameter t of the exponent of the proper clock $e^{-itH_{n\ell}}$ of that local system L_O (Definitions 1–3). Then R^6 can be regarded as R^4 with coordinates $(t, x) = (t_{(H_{n\ell},\mathcal{H}_{n\ell})}, x_{(H_{n\ell},\mathcal{H}_{n\ell})})$.

In this way we have two different frameworks or coordinate systems for the observable classical world and unobservable quantum world, respectively. These coordinate systems are 'orthogonal' in the vector bundle† $X \times R^6$ or $X \times R^4$, and coincide with each other at the center of mass of the observer's system. Therefore there is no contradiction between classical relativistic theory and quantum mechanical theory, even though the former is set on the curved Riemannian space and the latter is on the Euclidean space.

We call the Euclidean coordinates $(t, x) = (t_{n\ell}, x_{n\ell}) = (t_{(H_{n\ell},\mathcal{H}_{n\ell})}, x_{(H_{n\ell},\mathcal{H}_{n\ell})})$ inside the local system $L = (H_{n\ell}, \mathcal{H}_{n\ell})$ the **proper coordinate system**, and the curved Riemannian coordinates associated with the observer $L = (H_{n\ell}, \mathcal{H}_{n\ell})$ the **observer's coordinate system**.

The curved Riemann space appears in this way only related with the observation, and the Euclidean space appears as the framework of the unobservable inner quantum mechanical world.

For the observable classical world, we assume the following Axioms 4–5. We use the same notation $(t, x) = (t_{(H_{n\ell},\mathcal{H}_{n\ell})}, x_{(H_{n\ell},\mathcal{H}_{n\ell})})$ also to denote the classical coordinates in X for the observer's system $(H_{n\ell}, \mathcal{H}_{n\ell})$, for the difference between the classical and quantum coordinates are only in their metric.

General principle of relativity:

Axiom 4. Those laws of physics which control the **relative** motions of the **centers** of mass of the **observed** local systems are expressed by the classical equations which are covariant under the change of **observer's** coordinate systems of R^4:

$$(t, x) = (t_{(H_{mk},\mathcal{H}_{mk})}, x_{(H_{mk},\mathcal{H}_{mk})}) \to (t, x) = (t_{(H_{n\ell},\mathcal{H}_{n\ell})}, x_{(H_{n\ell},\mathcal{H}_{n\ell})})$$

†Here $X \times R^6$ is a trivial vector bundle with base space X and fibre R^6. Thus this vector bundle can be identified with the direct product of a Riemannian manifold X and a Euclidean space R^6 consisting of the pairs (x, u) with $x \in X$ and $u \in R^6$. More exactly, the vector bundle in the present case is a continuous mapping $\pi : X \times R^6 \to X$ such that $\pi^{-1}(x) = \{x\} \times R^6 \cong R^6$ for all $x \in X$.

for any pairs (m,k), (n,ℓ).

It is included in this axiom that one can observe the positions of other systems (i.e., their centers of mass) in his coordinate system (t,x).

The relative velocities of the observed systems are then defined as quotients of the relative positions of those systems and the (local and quantum mechanical) time t of the observer's system. These are our definitions of the measurement procedure of **classical** quantities, which accord with the ordinary (implicit) agreement among physicists where the time is given *a priori*.

Principle of equivalence:

Axiom 5. The coordinate system $(t_{(H_{n\ell},\mathcal{H}_{n\ell})}, x_{(H_{n\ell},\mathcal{H}_{n\ell})})$ associated with the local system $(H_{n\ell},\mathcal{H}_{n\ell})$ is the local Lorentz system of coordinates. Namely the gravitational potentials $g_{\mu\nu}$ for the **center** of mass of the local system $(H_{n\ell},\mathcal{H}_{n\ell})$, **observed** in this coordinates $(t_{(H_{n\ell},\mathcal{H}_{n\ell})}, x_{(H_{n\ell},\mathcal{H}_{n\ell})})$, are equal to $\eta_{\mu\nu}$. Here $\eta_{\mu\nu} = 0$ $(\mu \neq \nu)$, $= 1$ $(\mu = \nu = 1,2,3)$, and $= -1$ $(\mu = \nu = 0)$.

We do not assume the so-called field equation which determines the metric $g_{\mu\nu}$. We hold the room for the equations which would be found preferable in the future to the present ones.

Let us remark that the difference between the proper coordinate system and the observer's coordinate system is their metric. However at the center of mass of a local system L, these coincide with each other. In fact, at the center of mass of L, the Riemann metric $g_{\mu\nu}$ is equal to $\eta_{\mu\nu}$ by Axiom 5. Thus at any time t, the Riemann distance at the origin = the center of mass is given by

$$d\tau^2 = -g_{\mu\nu}(t,0,0,0)dx^\mu dx^\nu = -\eta_{\mu\nu}dx^\mu dx^\nu = dt^2 - dx_1^2 - dx_2^2 - dx_3^2.$$

For a person at the center of mass, $x = 0$ always. Thus for him or her

$$d\tau^2 = dt^2.$$

In this sense, the local time t is identified with the relativistic proper time τ.

The Euclidean distance inside the local system L is

$$d\ell^2 = dt^2 + dx_1^2 + dx_2^2 + dx_3^2.$$

Thus for the center of mass,

$$d\ell^2 = dt^2$$

again.

We set these two metrics on X and on R^4 so that they coincide with each other at the center of mass = the origin of both coordinate systems. These metrics do not contradict each other, for the spaces X and R^4 where these metrics are set are mutually orthogonal.

Summing up, we have

Theorem 2. Axioms 1–5 are consistent.

4 Observation

Given the system $(H_{mk}, \mathcal{H}_{mk})$ with coordinates $(t_{mk}, x_{mk}) = (t_{(H_{mk},\mathcal{H}_{mk})},\ x_{(H_{mk},\mathcal{H}_{mk})})$, we start with the quantum mechanics, i.e., with the Schrödinger propagator $e^{-it_{mk}H_{mk}}$. Then the quantum mechanical velocities of the particles in the system $(H_{mk}, \mathcal{H}_{mk})$ are given by the quotients $q_b = x_b/t_{mk}$, asymptotically as $t_{mk} \to \infty$, of the position vectors x_b of the particles and the local time t_{mk}.

Assumption on observation:

Axiom 6. The momenta $p_j = m_j x_j/t_{mk}$ of the particles j with mass m_j in the observed local system $(H_{mk}, \mathcal{H}_{mk})$ with coordinate system (t_{mk}, x_{mk}), given as above, are observed, by the observer system $(H_{n\ell}, \mathcal{H}_{n\ell})$ with coordinate system $(t_{n\ell}, x_{n\ell})$, as $p'_j = m_j x'_j/t_{n\ell}$, where x'_j is obtained from x_j by the relativistic transformation of coordinates: (t_{mk}, x_{mk}) to $(t_{n\ell}, x_{n\ell})$ as in Axiom 4. The same is true for the observation of the energies of the particles: the energies of the particles in the observed local system are observed by the observer as the ones transformed in accordance with the relativity.

Namely it is assumed that the quantum mechanical momenta $p_j = m_j x_j/t_{mk}$ of the particles within the system $(H_{mk}, \mathcal{H}_{mk})$ are observed in actual experiments by the observer system $(H_{n\ell}, \mathcal{H}_{n\ell})$ with coordinate system $(t_{n\ell}, x_{n\ell})$, as the **classical** quantities $p'_j = m_j x'_j/t_{n\ell}$ whose values are calculated or predicted by correcting the quantum mechanical values p_j with taking the relativistic effects of observation into account. A similar assumption is made for the energies of the particles.

Theorem 3. Axiom 6 is consistent with Axioms 1–5.

Indication of Proof: Axiom 6 is concerned only with the quantum mechanics **within** the local system $(H_{mk}, \mathcal{H}_{mk})$ so that it gives the rules to transform the **quantum mechanical** values, e.g., p_j, of the system $(H_{mk}, \mathcal{H}_{mk})$ to the **classical mechanical** values, e.g., p'_j, that would be **observed** experimentally by the observer. It is therefore not related with **any** physics laws of the particles **within** the system $(H_{mk}, \mathcal{H}_{mk})$, unless the transformed values (e.g., p'_j) are compared with the actual experimental values.

In this sense, Axiom 6 is concerned only with **how the nature looks to the observer**. Together with Axioms 1–5, it gives the prediction of the physical values observed in actual experiments, and is checked solely through the experimental data.

5 Scattering of one neutron in a uniform gravitational field

Experiment by Collela, Overhauser, and Werner [4]:

Consider the experiment done by Collela et al. [4] of measuring the interference of one neutron. This experiment is described in some simplification as in the following Figure 1:

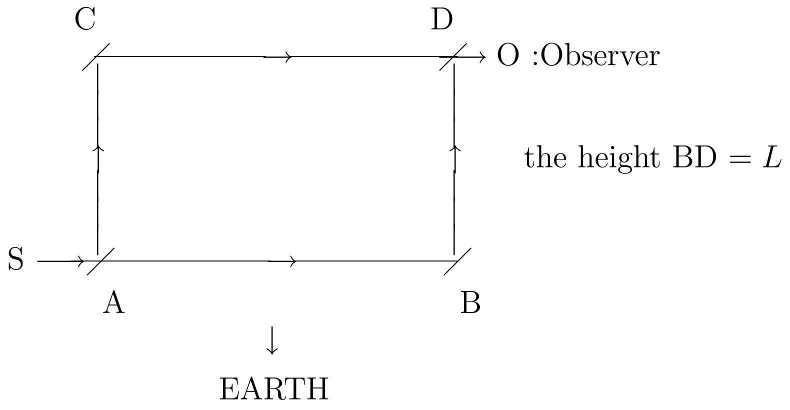


Figure 1

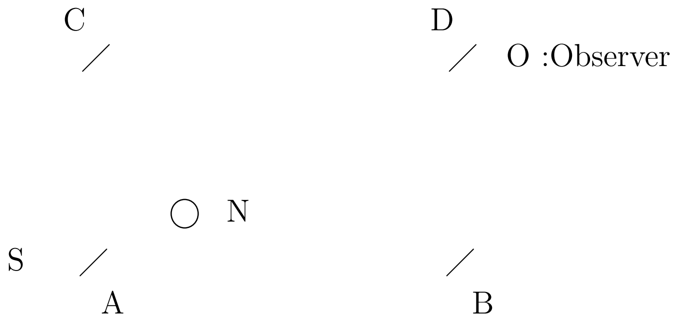
A neutron beam emitted at S is split into two beams by an interferometer at A, and the two beams are recombined at point D by other interferometers or mirrors B and C. The height L of the line BD on the earth can be varied. The dependence of the relative phase on L is given as follows, according to the experiment of [4] up to errors of about 1 %:

$$\hbar^{-1} mgLT, \tag{5.1}$$

where m is the mass of the neutron, g is the acceleration of gravity, and T is the (observed) time that the beams travel from C to D or A to B. This experiment shows that quantum mechanics and gravity play an important role *simultaneously* in the size of desktop environment. In fact, the lengths of the lines AB and BD are less than 10 cm in [4].

Explanation in our theory:

This experiment can be explained in our context, if we see it as a 3-body scattering phenomenon of a neutron N by two mirrors B and C as in Figure 2.



$$\downarrow$$

EARTH

Figure 2

Let the masses of mirrors B, C be M, the neutron mass be m, and $0 < m << M$. Then the Hamiltonian of this system is

$$H = \frac{p^2}{2m} + \frac{P_B^2}{2M} + \frac{P_C^2}{2M},$$

where p, P_B and P_C are the momentum operators for N, B and C. To separate the center of mass, we introduce the Jacobi coordinates with letting x, X_B and X_C the coordinates of N, B and C,

$$\begin{aligned} x_1 &= x - X_C, \\ x_2 &= X_B - \frac{mx + MX_C}{m + M}, \end{aligned} \tag{5.2}$$

or another Jacobi coordinates

$$\begin{aligned} x_1 &= x - X_B, \\ x_2 &= X_C - \frac{mx + MX_B}{m + M}. \end{aligned} \tag{5.3}$$

Using these coordinates, H can be written in the same form for both coordinates:

$$\begin{aligned} H &= H_1 + H_2, \\ H_1 &= \frac{p_1^2}{2\mu}, \quad H_2 = \frac{p_2^2}{2\nu}. \end{aligned}$$

Here note that the variables p_1 and p_2 are mutually independent, hence H_1 commutes with H_2, where p_1 and p_2 are the momenta conjugate to x_1 and x_2, and μ, ν are the reduced masses:

$$\mu^{-1} = m^{-1} + M^{-1}, \quad \nu^{-1} = M^{-1} + (m + M)^{-1}.$$

In the following, we denote the Hamiltonians for (5.2) by H_1, H_2, and for (5.3) by H_1', H_2'.

We take the unit system with $\hbar = h/(2\pi) = 1$. Then the propagation of the 3-body system is given by

$$\exp(-itH)f = \exp(-itH_1)\exp(-itH_2)f, \tag{5.4}$$

where $f(x) = f(x_1, x_2)$ is the initial wave function at the time $t = 0$, just after the neutron has been split into two beams by the interferometer A.

Remark. Here the **time** t is the local time determined by the Hamiltonian H or the correspondent local system, which we will denote by H in the sequel.

The decomposition (5.4) has two forms according to the choice of coordinates (5.2) or (5.3). (There is another choice, but it has nothing to do with our argument.)

The initial wave function $f(x_1, x_2)$**:** x_1 is the distance vector between N and C, or, between N and B, and x_2 is the distance vector between B and the center of mass of the system N+C, or between C and the center of mass of the system N+B. Therefore, as seen from the formula for x_2 in (5.2) or (5.3), we may regard it as

$$x_2 = X_B - X_C \quad \text{or} \quad X_C - X_B,$$

for M is larger enough than m. Thus we can regard x_2 as constant during the scattering process, hence $f(x_1, x_2)$ can be regarded as a function of x_1 only.

Namely $f(x_1, x_2)$ can be regarded as the wave function of the neutron N, and is split into two wave packets $f_1(x_1, x_2)$, $f_2(x_1, x_2)$ at time $t = 0$ by the interferometer A:

$$f = f_1 + f_2.$$

f_1 is the packet moving to the direction from A to C, and f_2 is the one from A to B.

Therefore (5.4) can be rewritten as follows:

$$\exp(-itH)f = \exp(-itH_1)\exp(-itH_2)f_1 + \exp(-itH_1')\exp(-itH_2')f_2. \tag{5.5}$$

As remarked above, we can regard $x_2 = X_B - X_C$ or $X_C - X_B$, therefore we can think $H_2 = H_2'$. Thus, noting that H_2 commutes with H_1 and that H_2' commutes with H_1', we have

$$(5.5) = \exp(-itH_2)\{\exp(-itH_1)f_1 + \exp(-itH_1')f_2\}.$$

The description up to here is by the local time t determined by the local system H.

The decomposition within { } of (5.5) gives a decomposition of the local system H into two local systems H_1 and H_1'. The reason that we could use the same local time t in these systems is that we were considering the scattering in the same local system H.

If there is no relativistic correlation between these local systems and the observer, the observer's time is the same as these local systems' times, and the observer sees the same phenomena as the ones with letting the observer's time t_O equal to the local time t of that local system H. Namely in this case, two different observations cannot distinguish the two local times.

However, if the gravitational field exists as in the present case, these local times can be distinguished by observation as follows.

H_1' is the Hamiltonian consisting of N and B, and its center of mass is regarded as located at B by $m << M$. Therefore that local system has a lower gravitational potential in amount gL compared to the observer O, hence the local time t of the local system H_1' is related with the observer's time t_O as follows approximately:

$$t = \frac{t_O}{\sqrt{1 + (2gL)/c^2}} = t_O(1 - (gL)/c^2).$$

Therefore
$$\exp(-itH_1')f_2 = \exp(-it_O \cdot H_1')\exp(it_O \cdot (gL/c^2)H_1')f_2.$$

H_1 is the local system consisting of N and C, and the center of mass is located at C with the same height as the observer. Hence
$$t = t_O.$$
We note that we can regard $H_1 = H_1'$, for $H = H_1 + H_2 = H_1' + H_2'$ and $H_2 = H_2'$.

From these, we have the following decomposition of the observational wave function for this 3-body system:
$$\exp(-itH)f = \exp(-itH_2)\exp(-it_O \cdot H_1)\{f_1 + \exp(it_O \cdot gLm)f_2\}. \tag{5.6}$$
Here we regarded f_1 as the wave function of the neutron N, and approximated its energy by the classical energy mc^2.

If we calculate the asymptotic behavior as $t \to \infty$ of the first two factors of (5.6), we have with $\mathcal{F}$ being the Fourier transformation
$$\begin{array}{rcl} (\exp(-itH_2)g)(x_2) & \to & C(t)\exp(itp_2^2/(2\nu))(\mathcal{F}g)(p_2) \quad (p_2 = x_2/t), \\ (\exp(-it_O \cdot H_1)g)(x_1) & \to & C(t)\exp(it_O \cdot p_1^2/(2\mu))(\mathcal{F}g)(p_1) \quad (p_1 = x_1/t_O), \end{array}$$
where $C(t)$ is the constant such that the absolute value of the first two factors in (5.6) equals 1 asymptotically as $t \to \infty$. Therefore there remains only the absolute value of the parentheses $\{\ \}$ of (5.6), which gives the desired phase difference and explains the interference observed in [4].

Remarks.

1. In the above the neutron N is regarded as moving from A to D in a classical velocity (v_1, v_2). Therefore the time necessary for N to reach D from A is given by $T = L/v_2$, which coincides with the time $T =$ the length of AB$/v$, where v is the horizontal velocity $= v_1$.

2. In the above scattering process, the interactions between N and B, C are not included in the Hamiltonian. This point may be remedied by introducing the very short-range potentials effective only in the vicinity of the neutron and the mirrors B, C. Actually these mirrors consist of a huge number of particles and the phenomenon should be treated as a many body problem including such a huge number of particles. But the above idealization works well for explaining the phenomenon.

6 Hubble's law

Hubble's law is a phenomenon that appears when one observes the light emitted from stars and galaxies far away from the earth. The emission of light itself is a quantum mechanical phenomenon that could be explained in the nonrelativistic quantum field theory as in [10], Section 11-(2). The observation or reception of this emission of light on the earth is explained as a classical observation according to our postulate Axiom 6, with assuming the Robertson-Walker metric as usual.

Robertson-Walker metric is the metric derived from the assumptions of *homogeneity* and *isotropy* of the large scale structure of the universe. We refer the reader to [12], Chap. 27 for details, and we here only outline the argument.

Under the hypotheses of homogeneity and isotropy, the metric is in general of the form

$$ds^2 = -(dx^0)^2 + d\sigma^2 = -(dx^0)^2 + a(x^0)^2\gamma_{ij}(x^k)dx^i dx^j,$$

where x^0 is the time parameter that 'slices' the spacetime by means of a one parameter family of some spacelike surfaces, and (x^1, x^2, x^3) is the 'comoving, synchronous space coordinate system' for the universe, in the sense of [12], sections 27.3–27.4. $a(x^0)$ is the so-called "expansion factor" that describes the ratio of expansion of the universe in the usual context of general relativity. A consideration by the use of homogeneity and isotropy yields ([12], section 27.6) that for some functions $f(r)$ and $h(x^0)$

$$ds^2 = -(dx^0)^2 + e^{f(r)}e^{h(x^0)}\{(dx^1)^2 + (dx^2)^2 + (dx^3)^2\}.$$

Assuming Einstein field equation $G^\mu{}_\nu - \lambda\delta^\mu{}_\nu = \kappa T^\mu{}_\nu$ and calculating, we get with replacing $e^{h(x^0)}$ by a constant times $e^{h(x^0)}$

$$ds^2 = -(dx^0)^2 + e^{h(x^0)}\left(1 + k\frac{r^2}{4r_0^2}\right)^{-2}(dx)^2,$$

where $k = 0$ or $+1$ or -1. This is called Robertson-Walker metric. Using the polar coordinates (r, θ, φ) and setting

$$\frac{r}{r_0} = u, \quad R(t) = r_0 e^{h(x^0)/2} \quad (t = x^0),$$

one can rewrite ds^2 as follows:

$$ds^2 = -(dt)^2 + R(t)^2\left(1 + \frac{k}{4}u^2\right)^{-2}[du^2 + u^2\{(d\theta)^2 + (\sin\theta d\varphi)^2\}].$$

Suppose $k = +1$, and consider a 3-dimensional sphere of radius A in a 4-dimensional Euclidean space

$$A^2 = (y^4)^2 + \sum_{k=1}^{3}(y^k)^2.$$

The metric on this sphere is

$$d\sigma^2 = \sum_{k=1}^{3}(dy^k)^2 + (dy^4)^2.$$

This is rewritten by using the above equation of the sphere as follows:

$$d\sigma^2 = \sum_{k=1}^{3} (dy^k)^2 + \left\{ A^2 - \sum_{k=1}^{3} (y^k)^2 \right\}^{-1} \left(\sum_{\ell=1}^{3} y^\ell dy^\ell \right)^2 .$$

Set $\rho^2 = \sum_{k=1}^{3} (y^k)^2$, and define v by

$$\rho = A \left(1 + \frac{v^2}{4} \right)^{-1} v.$$

Using polar coordinates (ρ, θ, φ) instead of (y^1, y^2, y^3), and rewriting ρ by the use of v, we have

$$d\sigma^2 = A^2 \left(1 + \frac{v^2}{4} \right)^{-2} [(dv)^2 + v^2\{(d\theta)^2 + (\sin\theta d\varphi)^2\}].$$

If we set $A = R(t)$, and identify v as u, this formula coincides with the space part $d\sigma^2$ of the above Robertson-Walker metric ds^2.

In this sense, the space part slice t = constant of the spacetime can be regarded as a 3-dimensional sphere of radius $R(t)$ in a 4-dimensional Euclidean space, hence $R(t)$ can be regarded as the radius of the universe.

In this context, the universe can be regarded as expanding when it is observed. Further the Hubble's cosmological redshift is explained in this context of classical observation also in our theory owing to Axiom 6, as in section 29.2 of [12].

We remark that the 'expansion' in this classical sense is different from the stationary universe ϕ in our context of quantum mechanical sense. The former 'expansion' is the result of an observation activity with fixing one observer's coordinate system, e.g., in the above explanation we have assumed a synchronous coordinate system, which explains why the universe looks expanding for all observers. The latter quantum mechanical stationary universe ϕ is the inner structure of its own and is independent of the observer's coordinate system. Theorem 2 guarantees that these two views are consistent with each other, and Axiom 6 predicts that this framework would explain and resolve the problems related with the actual observations. In the present problem of Hubble's law and 'expansion' of the universe, these phenomena are the consequences of the **observation** with one coordinate system fixed. In other words, they are 'appearance,' so to speak, which the universe makes under the 'interference' of the observer to try to reveal its figure or shape. More philosophically, the past or the future does not exist unless one fixes the time coordinate. The 'Big Bang' is an imagination under this **assumption** of the *a priori* existence of time coordinate. Unless it is observed with assuming the existence of a time coordinate, the universe is no more than a stationary state, which does not change and is correlated within itself as a whole.

Our theory is a reflection and a clarification of this assumption of the existence of time, adopted **implicitly** in almost all physics theories today.

Example of the last section is an experiment of human size, and the one in this section is an observation of cosmological size. These two examples would indicate an applicability of our theory to a unified treatment of physical phenomena of both sizes.

7 Uncertainty of time

According to Dereziński [5], one has for $f \in L^2(R^{3n})$

$$\int_1^\infty t^{-1} \left\| \left| q_a - \frac{x_a}{t} \right|^{1/2} J_a \left(\frac{x}{t} \right) h(H) e^{-itH} f \right\|^2 dt < \infty, \tag{7.1}$$

where a is a cluster decomposition, $h \in C_0^\infty(R^1)$, and J_a is a family of functions that gives a decomposition of configuration space.

This formula means roughly that

$$\begin{aligned} \left\| \left| q_a - \frac{x_a}{t} \right|^{1/2} J_a \left(\frac{x}{t} \right) e^{-itH} f \right\| &\leq C_f \quad \text{(uniformly in } t\text{)}, \\ &\to 0 \quad \text{(along some sequence } t = t_k \to \infty\text{)}. \end{aligned}$$

Rewriting this for the 2-body case by rereading the proof of Dereziński, one has

$$\left\| \left(\frac{x}{t} - q \right) e^{-itH} f \right\| \leq C_f, \quad \to 0 \text{ as } t = t_k \to \infty. \tag{7.2}$$

Let us calculate the uncertainty of time by using this relation.

First let us review the usual uncertainty relation. We consider the 2-body 1-dimensional case for simplicity. From

$$[p, x] = \frac{h}{2\pi i} I =: -iaI, \quad a \neq 0,$$

we have

$$2\mathrm{Im}(pf, xf) = a\|f\|^2.$$

Then by $a \neq 0$

$$0 \leq \|f\|^2 = \frac{2}{a}\mathrm{Im}(pf, xf) \leq \frac{2}{|a|}|(pf, xf)| \leq \frac{2}{|a|}\|pf\|\|xf\|.$$

Therefore for a normalized f with $\|f\| = 1$

$$\|pf\|\|xf\| \geq \frac{|a|}{2}.$$

Set now

$$\rho = (pf, f), \quad \sigma = (xf, f).$$

Then applying the above calculation to $p - \rho$, $x - \sigma$, one has the usual uncertainty relation

$$\Delta p \cdot \Delta x := \|pf - \rho f\|\|xf - \sigma f\| \geq \frac{|a|}{2} = \frac{h}{4\pi}.$$

If we note $p = mq$ (m is the reduced mass of the present 2-body system), we get

$$\Delta x \cdot \Delta q \geq \frac{h}{4\pi m}. \tag{7.3}$$

(Notice that this holds even if one replaces f by $e^{-itH}f$:

$$\Delta(e^{itH}xe^{-itH}) \cdot \Delta(e^{itH}qe^{-itH}) \geq \frac{h}{4\pi m}. \quad)$$

Set

$$R(t) = \frac{x}{t} - q.$$

Then the above Dereziński's inequality becomes

$$\|R(t)e^{-itH}f\| \leq C_f, \quad \to 0 \text{ as } t = t_k \to \infty.$$

As an approximate interpretation of this inequality, we take the following

$$\left(e^{itH}\left(\frac{x}{t} - q\right)^2 e^{-itH}f, f\right) \approx 0.$$

Then this means

$$R(t) = \frac{x}{t} - q \approx 0 \quad \text{or} \quad \frac{x}{t} \approx q, \tag{7.4}$$

where we have omitted the propagators $e^{\pm itH}$ for simplicity. Namely the expected value of time t is given by

$$t = \frac{(xe^{-itH}f, e^{-itH}f)}{(qe^{-itH}f, e^{-itH}f)}.$$

Taking Δ's of (7.4), we have

$$\frac{\Delta x}{\Delta t} \approx \Delta q.$$

Therefore

$$\Delta t \approx \frac{\Delta x}{\Delta q} = \frac{\Delta x \cdot \Delta q}{(\Delta q)^2}.$$

From the uncertainty relation (7.3), this implies

$$\Delta t \geq \frac{h}{4\pi m}\frac{1}{(\Delta q)^2}. \tag{7.5}$$

Here $(\Delta q)^2 = (qe^{-itH}f, qe^{-itH}f) - (qe^{-itH}f, e^{-itH}f)^2 \to 0$ along $t = t_k \to \infty$. (Notice that this convergence is very slow usually, due to (7.1).)

Conversely,

$$\Delta t \approx \frac{(\Delta x)^2}{\Delta x \cdot \Delta q} \leq \frac{4\pi m}{h}(\Delta x)^2.$$

Here $(\Delta x)^2 = (xe^{-itH}f, xe^{-itH}f) - (xe^{-itH}f, e^{-itH}f)^2 = O(t^2)$.

Thus, in the usual experiment where $t < 1$ sec, it is expected that the uncertainty Δt of time is approximately of the order

$$c_1\frac{h}{4\pi m} \leq \Delta t \leq C_2\frac{4\pi m}{h}. \tag{7.6}$$

In this sense the uncertainty of time is proportional to m^{-1}.

For example, in the case of neutron

$$m = 167 \times 10^{-26}\text{grams}, \quad \hbar = 1.054 \times 10^{-27}\text{erg} \cdot \text{sec}$$

yield

$$\frac{h}{4\pi m} \approx 3 \times 10^{-4}\text{cm}^2 \cdot \text{sec}^{-1}\,.$$

The dimension of c_1 in (7.6) is sec^2 $\cdot$cm^{-2} by the definition of $(\Delta q)^2$ after (7.5). This implies that the lower bound of the left side of (7.6) is of order

$$c_1 \times 10^{-4}\,\text{sec}\,.$$

This is the case of 2-body system of neutron and another particle. (Remember that the time is not defined for one body system.) In the actual case, we include the macroscopic systems in the observed system, as the mirrors B, C in Section 5. Therefore the uncertainty of time becomes extremely low.

For example, for the system of 1 μ grams = 1×10^{-6} grams, we have

$$\text{uncertainty } = c_1 \times 10^{-24}\,\text{sec},$$

and for the system of 1 gram

$$\text{uncertainty } = c_1 \times 10^{-30}\,\text{sec}\,.$$

8 Concluding discussions

We summarize the framework of our theory as in [10], Section 10:

Local times:

- The times are defined only for local systems $(H_{n\ell}, \mathcal{H}_{n\ell})$.
- The total universe ϕ has no time associated.
- The local times arise through the affections from other particles outside the local systems. (Definitions 1–3.)
- The uncertainty principle holds only within these local systems as the uncertainty of the local times.
- The quantum mechanics is confined within each local system in this sense.
- The quantum mechanical phenomena between two local systems appear only when they are combined as a single local system.

- In the local system, the interaction and forces propagate with infinite velocity or in other words they are **unobservable**.

Local systems:

- Each local system can be the observer of other systems.
- In this situation the local systems are **mutually independent** in the sense that the associated quantum mechanical local times are not correlated in general.
- Therefore there are no reasons to exclude the classical mechanics in describing the **observable** relative behavior of the observed systems with respect to the observer.
- Thus the gravitational potentials can be introduced in accordance with the theory of general relativity.
- These potentials determine the global space-time structure around the observer system.
- Inside the observer system the space-time is Euclidean.
- The observer itself cannot detect the gravitational correlation or the space-time structure inside its own system.
- On the contrary, between the local systems, the observer can detect only the classical mechanical effects.
- Nevertheless, through the media (e.g., light in classical sense) which connect the observer and the observed systems and obey the classical physics, the observer sees, through some relativistic corrections of the observed classical values, that the physics laws inside the other local systems follow the quantum mechanics.

Total universe:

- These facts are all the consequences of the introduction of **local times** which are proper to each local system.
- The time is neither a given thing nor a common one to the total universe.
- On the contrary there can be defined no global time. More strongly the total universe is a (stationary) bound state of the total Hamiltonian H of infinite degrees of freedom.
- The times arise only when the observers restrict their attention to its subsystems as approximations of the total Hamiltonian H.
- The universe itself is correlated within it as a bound state of H.
- The observer always separates a subsystem from it, so to speak, artificially, and the (steady) motion and time appear.

- Inside the subsystem this local time explains the quantum effects, and outside the subsystem it explains the gravitation and the classical mechanics. The relativistic quantum phenomena are explained as the relativistic effects of the observation of the non-relativistic quantum systems.

- All these physical phenomena occur by this artificial separation of the universe. The universe itself does not 'change': It is a stationary bound state.

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